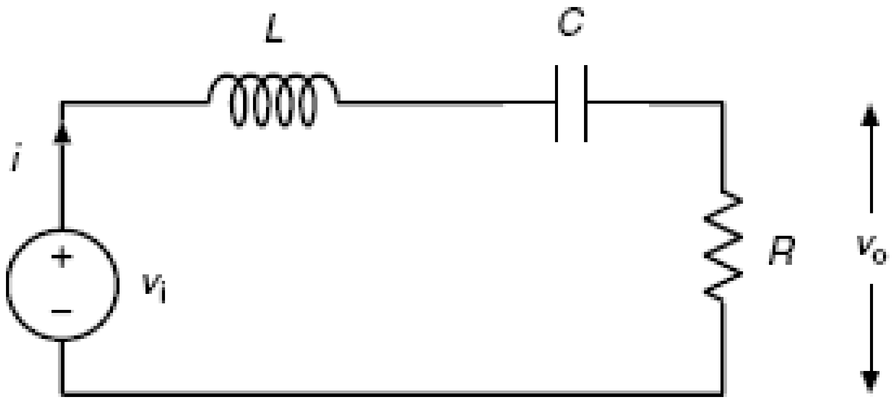
Frequency selective circuits and matching

- Frequency selective circuits are extremely important elements of an RF system. They often consist of a combination of inductors and capacitors that achieves maximum power transfer at a particular frequency or range of frequencies.
- Frequency selective circuits need not be restricted to combinations of inductors and capacitors, but can also consist of lengths of transmission line or electromechanical devices such as quartz crystals.
- Fundamental to selective circuits is the concept of *resonance*. That is, if a circuit is driven by an oscillatory stimulus, there will be a frequency, or frequencies, at which the circuit response peaks.

Series resonant circuits

- We first investigate the response of a series circuit to a voltage step: switched on at time $t = 0$



$$v_o(s) = \frac{s(R/L)}{s^2 + Rs/L + 1/LC} v_i(s) = A(s) v_i(s).$$

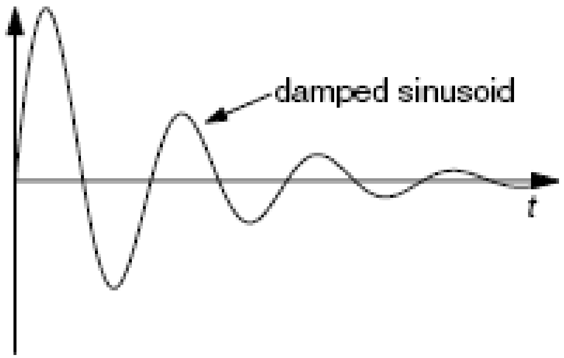
- for the voltage step: $v_i(s) = s^{-1}$ and hence

$$v_o(s) = \frac{R/L}{s^2 + sR/L + 1/LC} = \frac{RC}{s^2/\omega_0^2 + (2\zeta/\omega_0)s + 1},$$

- where $\omega_0 = 1/\sqrt{(LC)}$ is the *natural frequency* and $\zeta = (R/2)\sqrt{(C/L)}$ is the *damping ratio;* note that $v_o(s)$ has poles at $-\zeta\omega_0 \pm \sqrt{(\omega_0 \zeta^2 - 1)}$

Series resonant circuits

- If $\zeta > 1$, the poles are real and the inverse Laplace transform yields an exponential fall in v_o.
- If $\zeta < 1$, the roots form complex conjugate pairs with a non-zero imaginary part. In this case we have damped oscillation



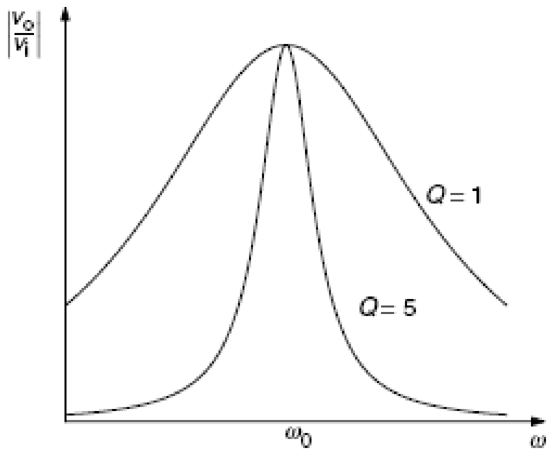
- Note that the circuit selects a particular frequency at which to oscillate, this selective property of tuned circuits is of prime importance in RF design

Series resonant circuits

- In the time harmonic case, we define the transfer function $A(j\omega)$ by:

$$A(j\omega) = \frac{1}{1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})},$$

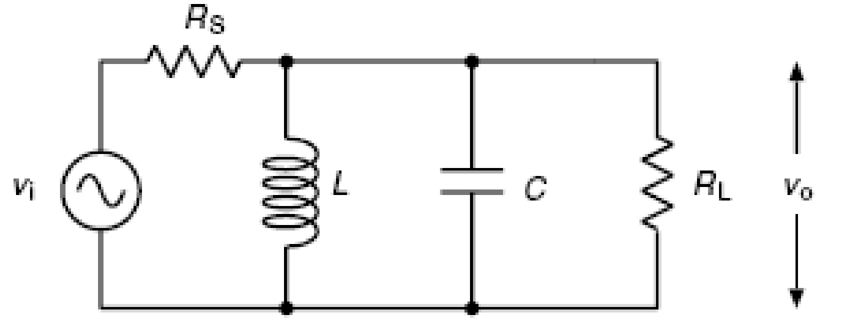
- where $Q = 1/\omega_0 RC = \omega_0 L/R = 1/2\zeta$. The maximum of v_o will occur at the resonant frequency ω_o $(= 1/\sqrt{LC})$



$$\omega_{3\,\mathrm{dB}} = \omega_0 \pm \frac{\omega_0}{2Q}$$

Parallel resonant circuits

- For a capacitor and inductor connected in parallel, there will be a frequency (the resonant frequency) at which the combination has infinite impedance.



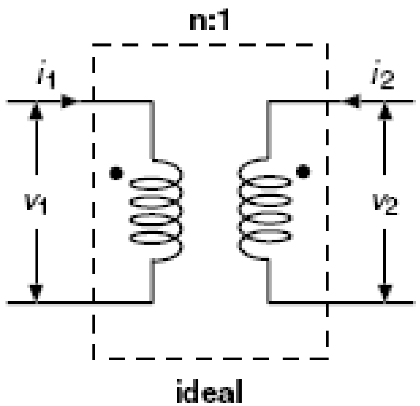
- The output voltage is given by:

$$v_o = \frac{Z}{R_S + Z} v_i = \frac{j\omega L R_L}{R_S R_L + j\omega L(R_S + R_L) - \omega^2 LC R_L R_S} v_i.$$

- Maximum output voltage $v_o = R_L v_i/(R_L + R_S)$ occurs at resonant frequency for which the combined impedance of L and C is infinite.

Inductive transformers

- Impedance transformation is an extremely important function to manipulate load characteristics in order to achieve maximum power transfer.
- At low frequencies, this is achieved through the mutual inductance of magnetically linked inductor windings. Under ideal circumstances, the device simply transforms voltage according the ratio of the windings and current according to the inverse of this ratio.
- Unfortunately, at radio frequencies, the self-inductance of a transformer becomes important and the action of the transformer needs to be considered in terms of a more complex model.



$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt},$$

$$v_1(s) - \frac{M}{L_2}v_2(s) = s\left(L_1 - \frac{M^2}{L_2}\right)i_1(s)$$

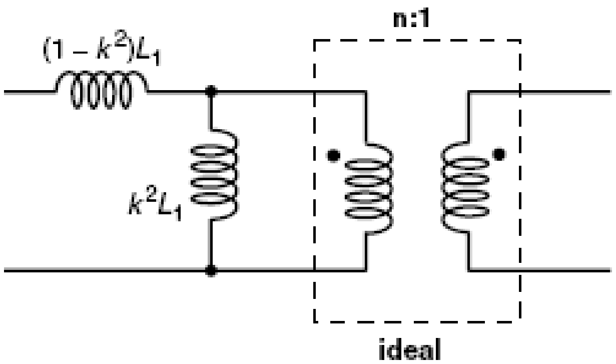
$$i_1(s) + \frac{M}{L_1}i_2(s) = \frac{v_1}{sL_1}.$$

Inductive transformers

- for L_2 L_1 infinite and $M=(L_1L_2)^{1/2}$, and $n = (L_1/L_2)^{1/2}$ we obtain an ideal transformer

$$i_1 = -\frac{i_2}{n} \quad \text{and} \quad v_1 = n v_2, \quad \longrightarrow \quad Z_1 = n^2 Z_2$$

- the deviation from the ideal is measured in terms of the size of $sL1$ and the coupling coefficient



$$k = \frac{M}{(L_1 L_2)^{\frac{1}{2}}}.$$

- The effective turns ratio *n is*

$$n = k \left(\frac{L_1}{L_2}\right)^{\frac{1}{2}}$$

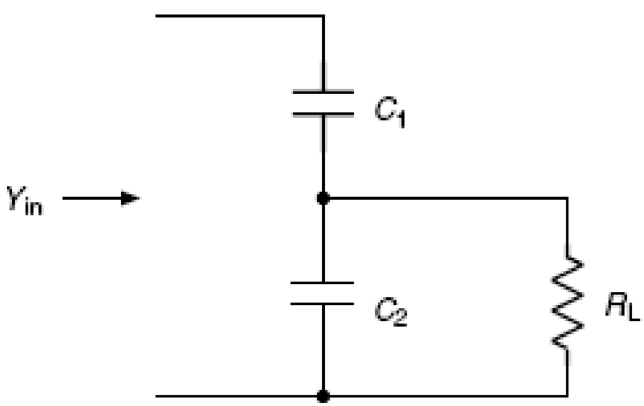
Capacitive transformers

- An alternative to the inductive transformer is the capacitive divider. The transformed impedance *Z*in will be equal to (1/j*ω**C*1) + (1/j*ω**C*2)||*R*L which implies the transformed admittance:

$$Y_{\rm in} = \frac{{\rm j}\omega C_1 - \omega^2 R_{\rm L} C_1 C_2}{{\rm j}\omega R_{\rm L}(C_1 + C_2) + 1} = G_{\rm in} + {\rm j}B_{\rm in}.$$

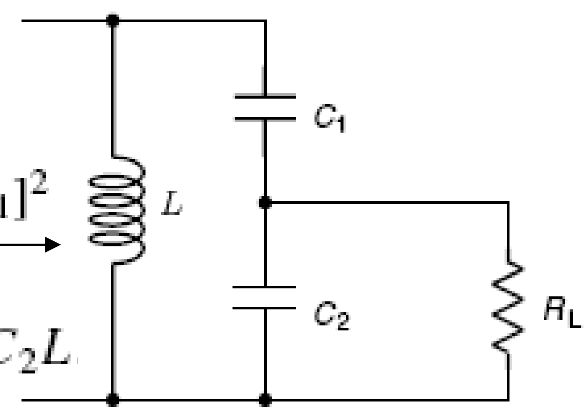
- if RL is much greater than C1 and C2:

$$G_{\rm in} \approx \frac{1}{R_{\rm L}}\left(\frac{C_1}{C_1 + C_2}\right)^2 \quad \text{and} \quad B_{\rm in} \approx \frac{\omega C_1 C_2}{C_1 + C_2}.$$

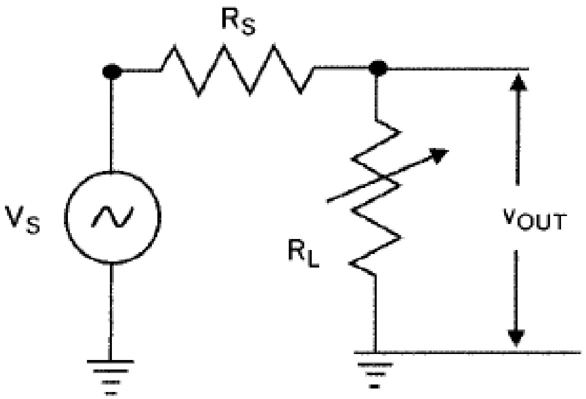


$$R_{\rm in} = R_{\rm L}[(C_1 + C_2)/C_1]^2$$

$$\omega_0^2 = (C_1 + C_2)/C_1 C_2 L$$

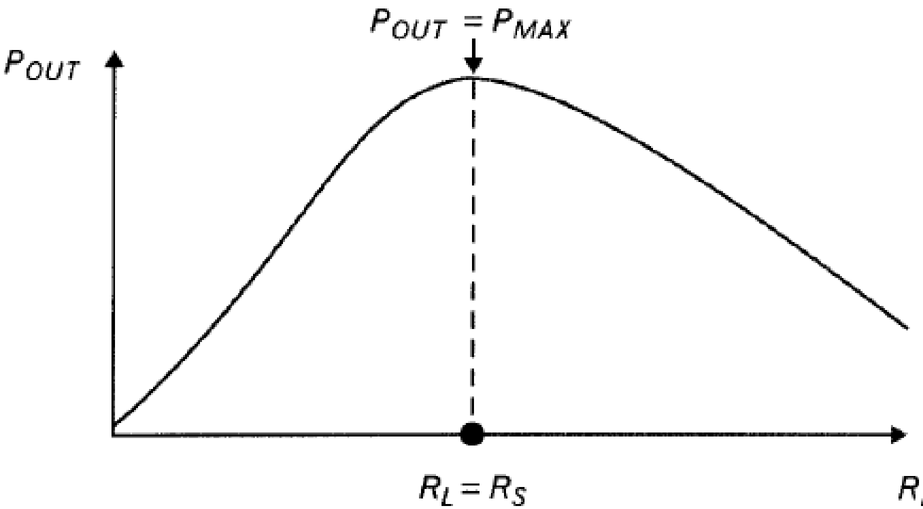


Impedance matching: maximum dissipated power



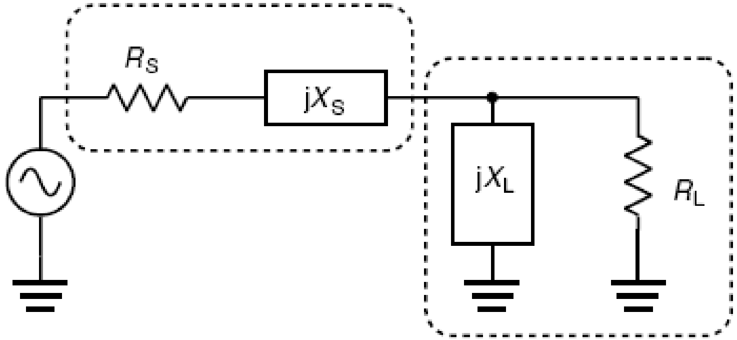
$$P_{OUT} = \frac{v_{OUT}^2}{R_L} = \frac{\left(\frac{R_L}{R_S + R_L}\right)^2}{R_L}\left(v_S^2\right)$$

$$= \frac{R_L}{\left(R_S + R_L\right)^2}\left(v_S^2\right)$$



L-network matching

- A means of achieving impedance transformation is through the L-network



- Consider the problem of matching source impedance R_S to load impedance R_L.
- The unknown reactances X_S and X_L need to be chosen such that R_L is transformed into R_S.
- We will select X_L such that $jX_L || R_L$ has resistive component R_S and reactive component $-X_S$, that is:

$$\frac{jX_L R_L}{R_L + jX_L} = \frac{X_L^2 R_L + jX_L R_L^2}{R_L^2 + X_L^2} = R_S - jX_S$$

L-network matching

$$R_S = \frac{X_L^2 R_L}{R_L^2 + X_L^2} \quad \text{and} \quad X_S = -\frac{X_L R_L^2}{R_L^2 + X_L^2}$$

$$R_S = \frac{R_L}{1 + Q^2} \quad \text{and} \quad X_S = -\frac{X_L Q^2}{1 + Q^2},$$

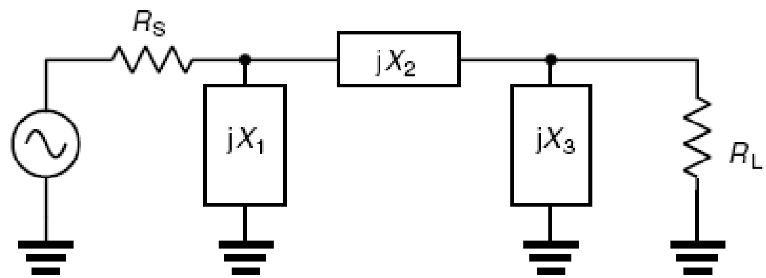
- where $Q = |R_L/X_L|$ is the Q of jX_L R_L. It will be noted that $Q = \sqrt{((R_L/R_S) - 1)}$.
- Consequently, to match source R_S to load R_L, calculate $Q = \sqrt{((R_L/R_S) - 1)}$ and then obtain the required reactances according to

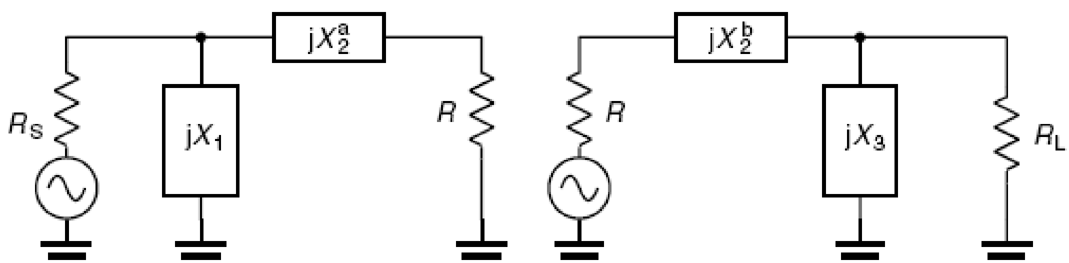
$$X_L = \pm\frac{R_L}{Q} \quad \text{and} \quad X_S = -\frac{X_L Q^2}{Q^2 + 1}.$$

- <u>Note that we have assumed that $R_L > R_S$. Otherwise, we will need to reverse the L-network</u>

π- and T-networks

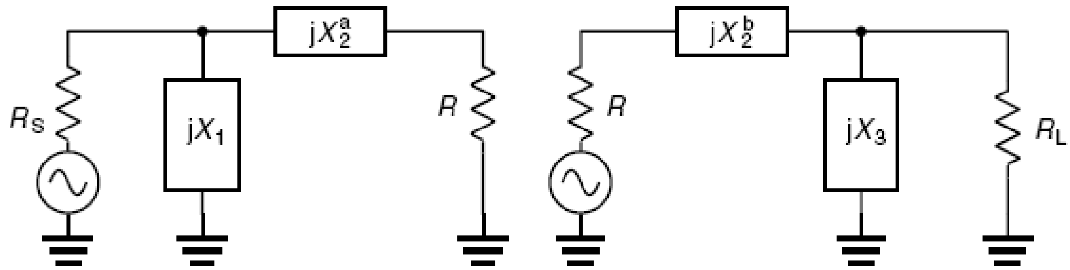
- In some circumstances it might also be desirable for the matching network to perform the function of a band-pass filter. In such cases, the *π*-network would be appropriate.
- Because of this common impedance, the L-networks can be joined directly to form a *π* network. Network values can be found by solving two equivalent problems
- we must have $R < R$S and $R < R$L





π- and T-networks

- For the calculation of bandwidth we take an overall quality factor $Q = \sqrt{[\max(R_S, R_L)/R] - 1}$, which is the highest Q of the two L-networks.
- R is chosen to achieve a given bandwidth (bandwidth = frequency/Q), but it should be noted that R can be no greater than $\min(R_S, R_L)$.
- Both R_S and R_L are matched to R and the required value of X_2 is calculated from $X_2^a + X_2^b$

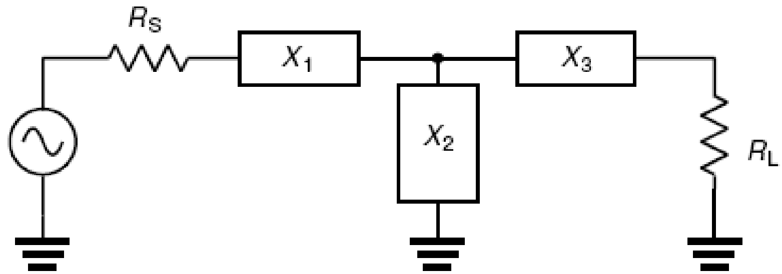


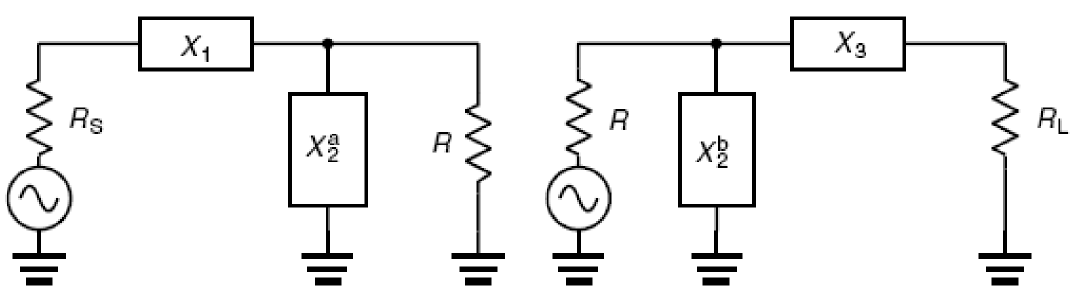
π- and T-networks

- An alternative is the T-network; L-networks are used to transform impedances R_S and R_L into a common impedance R, but with

$$R > R_S \quad \text{and} \quad R > R_L.$$

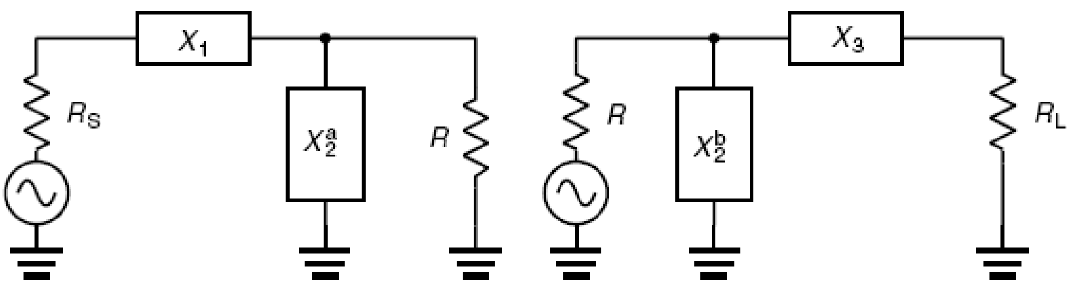
- Because of their common impedance, the L-networks can be joined directly to form a T-network. Network reactances are found by solving the two problems illustrated





π- and T-networks

- For design purposes we take an overall quality factor $Q = \sqrt{[R/\min(R_S, R_L)] - 1}$, which is the highest Q of the two L-networks.
- R is chosen to achieve a given bandwidth, but it should be noted that R must be greater than $\max(R_S, R_L)$.
- Both R_S and R_L are matched to R and the required value of X_2 calculated from $X_2 = X_2^a || X_2^b$.



L-network example

- A 100 MHz source with internal resistance Z_S = 25 + j5 ohm is to be matched to a 50 load by means of an L-network, the network is required to operate as a high-pass filter.

- X_S will need to be capacitive and X_L inductive for a high-pass characteristic;
- We will need $jX_L||50$ to have resistive component 25 and a reactive component that cancels 5 + X_S.
- Consequently:

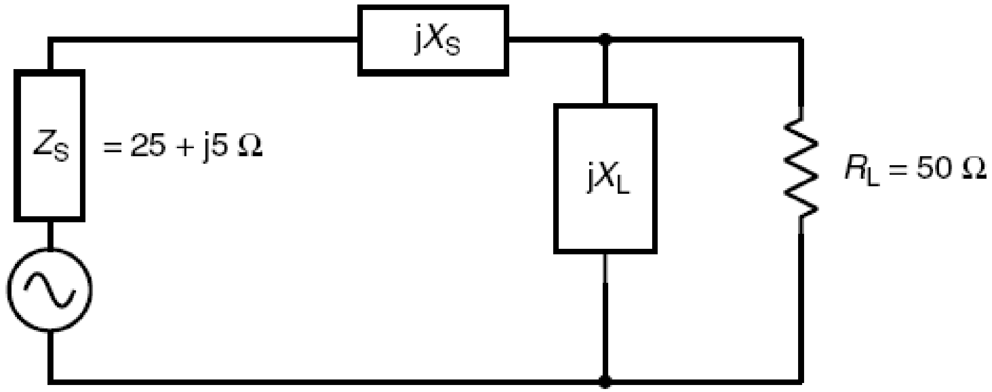
$$25 = \frac{X_L^2 R_L}{R_L^2 + X_L^2} \Rightarrow X_\mathrm{L} = 50\,\Omega \quad \text{and} \quad 5 + X_\mathrm{S} = -\frac{X_L R_L^2}{R_L^2 + X_L^2} \Rightarrow X_\mathrm{S} = -30\,\Omega$$

L-network example

- where XL has been chosen to be positive in order to generate a shunt inductance. Noting that $\omega = 628.3 \times 10^6$ rad/sec, the values of C and L corresponding to reactances XS and XL will be given by

$$C = -\frac{1}{\omega X_S} = \frac{1}{628.3 \times 10^6 \times 30} = 53\,\mathrm{pF}$$

$$L = \frac{X_L}{\omega} = \frac{50}{628.3 \times 10^6} = 79.6\,\mathrm{nH}.$$



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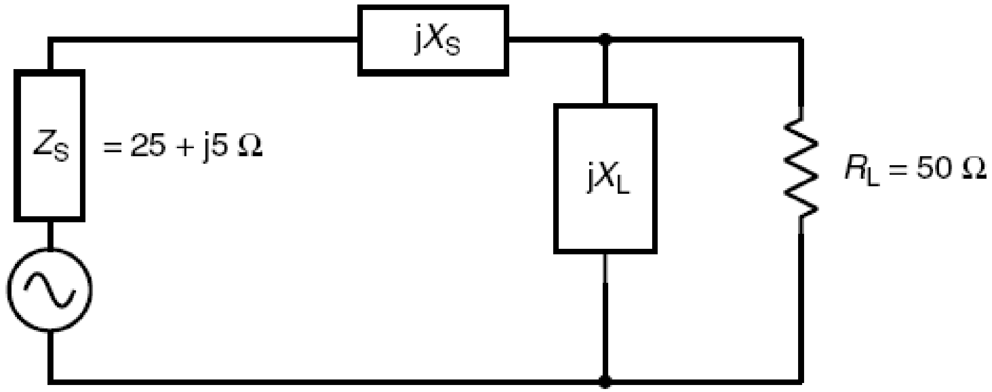
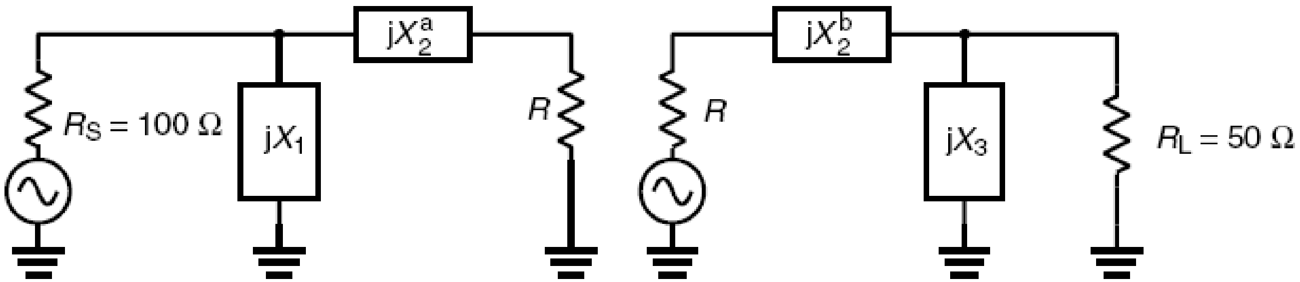


π-network example

- A 100 MHz source with 100 internal resistance is to be matched to a 50 *ohm* load by means of a *π*-network. The network is to exhibit a 3 dB bandwidth of 5 MHz.

- The problem is solved by converting into two auxiliary L-network matching problems
- we can find the values of *X*1, *X*2 and *X*3
- the value of *X*2 is obtained from *Xa*2+*Xb*2 after solving the auxiliary problems.
- For a 5MHz bandwidth at 100 MHz, we will need an overall *Q* of 20 (bandwidth = frequency/*Q*) and hence a value of 0.25 *ohm* for *R*



π-network example

- we choose $X1$ and $X3$ to be inductive. For the first network (a), $Q = \sqrt{(RS/R)-1}$ ≈ 20, so

$$X_1 = \frac{R_S}{Q} \quad \Rightarrow \quad X_1 \approx 5 \quad \text{and} \quad X_2^a = -\frac{X_1 Q^2}{Q^2+1} \quad \Rightarrow \quad X_2^a \approx -5$$

- For the second network (b), $Q = \sqrt{(RL/R)-1} \approx 14.1$, so

$$X_3 = \frac{R_L}{Q} \quad \Rightarrow \quad X_3 \approx 3.55 \quad \text{and} \quad X_2^b = -\frac{X_3 Q^2}{Q^2+1} \quad \Rightarrow \quad X_2^b \approx -3.55$$

- For the π-network we have $X1 = 5$, $X2 = -8.55$ and $X3 = 3.55$.
- Since $\omega = 628.3 \times 106$ rad/sec, we have $L1 = 7.95$ nH, $C2 = 186$ pF and $L3 = 5.65$ nH.
- It will be noted the filter has a band-pass characteristic